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LETTER TO THE EDITOR

Oscillations in the frequency dependence of long-range correlations of waves

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Abstract. We predict new oscillations in the frequency dependence of the intensity autocorrelation functions of waves for a tube geometry. We have performed numerical simulations which clearly confirm the existence of new long-range correlations.

Recently, much work has been devoted [1–7] to the study of intensity correlations of waves in random systems. Quite remarkably, these correlations are of a long-range nature and enhance [1] the fluctuations of the transmission coefficient in a non-classical manner [1,3]. That is, $\langle T^2\rangle - \langle T\rangle^2$ is not proportional to Ω^{-1} (where Ω is the volume) but depends explicitly on the length L (in a slab geometry) and the width W of the system.

In this letter, we study the frequency dependence of the intensity autocorrelation function $C(\Delta\omega)$ for a tube geometry, where $L \gg W$. Recently, this geometry has become interesting experimentally [6,7]. We find new behaviour for $C(\Delta\omega)$ for $\Delta\omega > D/W^2$ (where D is the diffusion constant). In this region, $C(\Delta\omega)$ oscillates as $\Delta\omega$ increases with an amplitude which decays slowly as $\Delta\omega^{-1}$. We have also performed numerical simulations by applying the Edrei–Kaveh [8] method to this new geometry and have found new long-range behaviour.

The long-range nature of the intensity fluctuations was first demonstrated for their angular dependence. This was achieved by two independent methods: by diagrammatic techniques [1,2] and by the Langevin approach [4]. Both methods seem to coincide (up to numerical factors [4]). The angular correlation function is defined as

$$C(\Delta\bar{q}_a, \Delta\bar{q}_b) \equiv \langle \delta I(\bar{q}_a, \bar{q}_b)\delta I(\bar{q}_{a'}, \bar{q}_{b'})\rangle \tag{1}$$

where $\delta I(\bar{q}_a, \bar{q}_b) = I(\bar{q}_a, \bar{q}_b) - \langle I(\bar{q}_a, \bar{q}_b)\rangle$ and similarly for $\delta I(\bar{q}_{a'}, \bar{q}_{b'})$. The vectors $\bar{q}_a, \bar{q}_b$ correspond to the incident and emitted wavevector, respectively. For two given such pairs of wavevectors $(\bar{q}_a, \bar{q}_b)$ and $(\bar{q}_{a'}, \bar{q}_{b'})$, it was shown that C is a function of $\Delta\bar{q}_a = \bar{q}_a - \bar{q}_{a'}$ and $\Delta\bar{q}_b = \bar{q}_b - \bar{q}_{b'}$. Equation (1) can be expanded [2] in powers of the inverse dimensionless conductance $g^{-1} = (\lambda/W)^{d-1}(L/l)$, where l is the elastic transport mean free path and d is the dimensionality of the system. Thus, (1) can be written as

$$C(\Delta\bar{q}_a, \Delta\bar{q}_b) = \sum_n g^{1-n}C_n(\Delta\bar{q}_a, \Delta\bar{q}_b). \tag{2}$$

Until now, only the first three terms have been calculated. For wide samples for which $W \gg L$, g^{-1} is extremely small which causes C_2 and C_3 to be almost unobservable. The short-range contribution C_1 is called the 'memory effect' [2,9,10]. C_1 was readily determined numerically [5] and observed experimentally [10]. The main point to note is that C_2 is identical for a two-dimensional system and a three-dimensional system. It is therefore easier to detect C_2 for a two-dimensional system because g^{-1} is then larger by a factor W/λ. Indeed, $C_2(\Delta\bar{q}_a, \Delta\bar{q}_b)$ has recently been obtained from numerical simulations [11] and found to be in agreement with the analytical predictions. C_3 has not yet been determined for a wide slab for which $W \gg L$.

By analogy with (1), one may define the frequency-dependent autocorrelation function

$$C(\Delta\omega) = \langle \delta I(\omega)\delta I(\omega + \Delta\omega)\rangle \tag{3}$$

where $\delta I(\omega) = I(\omega) - \langle I(\omega)\rangle$. This can also be expanded in powers of g^{-1},

$$C(\Delta\omega) = \sum_n g^{1-n} F_n(\Delta\omega). \tag{4}$$

For wide samples for which $W \gg L$, only $F_1(\Delta\omega)$ has been determined numerically [11] or experimentally [12]. The long-range correlations, $F_2(\Delta\omega)$ and $F_3(\Delta\omega)$ contribute negligibly to $C(\Delta\omega)$. Analytical expressions for $F_1(\Delta\omega)$ were obtained [13] for different geometries by showing [13] that

$$F_1(\Delta\omega) = |\textstyle\int P(t)\exp(\mathrm{i}\Delta\omega t)\,\mathrm{d}t|^2 \tag{5}$$

where $P(t)$ is the diffusive probability for a multiple-scattering trajectory of length Vt (where V is the velocity of light in the medium). Equation (5) has been obtained by Edrei and Kaveh [13] and confirmed by numerical simulations [8]. Genack and Drake determined [12] $P(t)$ experimentally from the transmitted pulse shape of a slab and showed that the resultant $F_1(\Delta\omega)$ (from (5)) is in excellent agreement with the measured $C(\Delta\omega)$. This confirms that the contributions of the higher-order terms in (4), $F_2(\Delta\omega)$ and $F_3(\Delta\omega)$, are negligible when $W \gg L$.

Recently, van Albada and Lagendijk [7] showed that when a point source is used instead of a plane wave, the contribution of $F_2(\Delta\omega)$ to the total transmission coefficient is enhanced. They were able to determine $F_2(\Delta\omega)$ and showed that it agrees with the calculation of Pnini and Shapiro [4].

Genack and co-workers [6] has recently pointed out that by using a tube geometry, one enhances the contributions of F_2 and F_3 because g^{-1} becomes rather larger when $W \ll L$.

The purpose of this letter is to show that the functional form for a tube geometry is entirely different from the wide geometry, for which $W \gg L$. We show that $F_2(\Delta\omega)$ depends markedly on W/L. For $W/L \ll 1$, we find that $F_2(\Delta\omega)$ is almost *independent* of $\Delta\omega$ and contributes a *constant* correlation, similar to F_3. Thus, in this regime F_2 is indistinguishable from the constant F_3 except that its contribution to $C(\Delta\omega)$ is larger by a factor g.

When $W \ll L$, we find for $F_2(\Delta\omega)$,

$$F_2(\Delta\omega) = \left(\frac{\sin x}{x}\right)^2 \tag{6}$$

where $x = (\Delta\omega/D)^{1/2}W/2$. We see that, unlike the case of a wide system $W \gg L$, $F_2(\Delta\omega)$ is *independent* of the length of the system L. This is in sharp contrast to $F_1(\Delta\omega)$ in this geometry. We can show that, to an excellent approximation, $F_1(\Delta\omega)$ is *independent* of W and continues to scale as $(\Delta\omega/D)^{1/2}L$. To prove this result, we turn to (5) and show that $P(t)$ is almost independent on W. For a given W, we have to impose an additional two reflecting boundary conditions (we use a two-dimensional tube). In this case, we get $P_w(t) = P(t)\Delta(t,y)$, where $P(t)$ corresponds to $W \to \infty$ and $\Delta(t,y)$ is the correction due to a finite W. In this case, $P_w(t)$ depends on the distance y from the side boundary. Solving the diffusion equation with the above boundary conditions yields

$$\Delta(t,y) = \sum_{n=0}^{\infty} n^{-1} \sin(2\pi n y_0/W) \cos(2n\pi y/W) \exp(-\pi^2 D n^2 t/W^2) \quad (7)$$

where y_0 is a negligibly small number ($y_0 \ll W$) which is introduced to avoid conflicting boundary conditions at the corners $y = 0$ and $y = W$. As long as $W > l$, we must have $y_0 \ll W$ and only the $n = 0$ term in (7) contributes, leading to $\Delta(t,y) = 1$. This result is in striking contrast to $C_1(\Delta\bar{q}_a, \Delta\bar{q}_b)$ for a tube geometry which was recently shown by Eliyahu *et al* [14] to differ significantly from a wide system where $W \gg L$. From this result we may conclude that even for a tube geometry (where g^{-1} is not small), $F_2(\Delta\omega)$ may be observed only for $\Delta\omega > D/L^2$, where $F_1(\Delta\omega)$ is small. For $\Delta\omega$ near the half-width $\Delta\omega_{\mathrm{HW}} = D/L^2$, we get for x in (6), $x = W/2L$. Since for a tube geometry $W/L \ll 1$, we find $F_2(\Delta\omega \lesssim D/L^2) \simeq 1$. Thus, $F_2(\Delta\omega)$ is almost constant and indistinguishable from F_3. When $\Delta\omega$ increases much above $\Delta\omega_{\mathrm{HW}}$, $F_1(\Delta\omega)$ decays exponentially and $F_2(\Delta\omega)$ is revealed. When $\Delta\omega > D/W^2$, $F_2(\Delta\omega)$ oscillates according to equation (6). Thus, the range of frequencies where these oscillations should be observed is $\Delta\omega \gtrsim \Delta\omega_{\mathrm{HW}}(L/W)^2$. We may distinguish two cases. For tubes where $L \gg W$, the oscillatory region is difficult to reach and $F_2(\Delta\omega) = 1$. For tubes where $L \gtrsim W$, we predict that near the tail of $C(\Delta\omega)$ (for $\Delta\omega > \Delta\omega_{\mathrm{HW}}$), oscillatory behaviour should set in.

We have performed numerical simulations to verify these predictions, using the Edrei–Kaveh method [8] for calculation $C(\Delta\omega)$. We first show the results for a two-dimensional tube with $L = 7W$. In this case, oscillatory behaviour should be observed only for $\Delta\omega > 49\Delta\omega_{\mathrm{HW}}$. In figure 1, we show our numerical results and compare them with the theory. All the results are normalized to $C(\Delta\omega = 0)$. The squares represent $F_1(\Delta\omega)$. It should be emphasized that $F_1(\Delta\omega)$ is calculated numerically directly from the electric field/electric field correlation, $F_1(\Delta\omega) \equiv |\langle E^*(\omega)E(\omega + \Delta\omega)\rangle|^2$. We compare our simulations with $F_1(\Delta\omega)$ as calculated from (5). The excellent agreement is evident from the figure. In figure 1, we also show the entire correlation function $C(\Delta\omega)$ which is represented by the plus signs. The fact that $C(\Delta\omega)$ is always larger than $F_1(\Delta\omega)$ is due to the long-range contributions $F_2(\Delta\omega)$ and $F_3(\Delta\omega)$. For this geometry, we plot $C_2(\Delta\omega) = g^{-1}F_2(\Delta\omega)$ as given by (5). For the range of frequencies in the figure, $F_2(\Delta\omega) = 1$ and $C_2(\Delta\omega) = g^{-1} = 0.28$. We also plot $C_3(\Delta\omega) = g^{-2}F_3(\Delta\omega)$. Since $F_3(\Delta\omega) = 1$, $C_3(\Delta\omega) = g^{-2} = 0.078$. The broken curve in figure 1 represents the total contribution $C(\Delta\omega) = C_1(\Delta\omega) + C_2(\Delta\omega) + C_3(\Delta\omega)$. We see that there is excellent agreement between the analytical results for $C(\Delta\omega)$ and those obtained by the simulations (the plus signs). Figure 1 confirms our prediction that for $L \gg W$, $F_2(\Delta\omega) = 1$ and is similar to $F_3(\Delta\omega) = 1$. Of course, the contribution of $F_2(\Delta\omega)$ to $C(\Delta\omega)$ is larger than $F_3(\Delta\omega)$ by a factor $g = 3.6$.

We now turn to shorter tubes in order to study the existence of the oscillations of $F_2(\Delta\omega)$. We have used a two-dimensional tube with $L = 2W$. Here we expect to see oscillations for $\Delta\omega \gtrsim 4\Delta\omega_{\mathrm{HW}}$. In figure 2, we plot $C(\Delta\omega)$ as a function of $\Delta\omega$. The broken curve is $F_1(\Delta\omega)$ which agrees with the simulations for $\Delta\omega < \Delta\omega_{\mathrm{HW}}$. For $\Delta\omega > \Delta\omega_{\mathrm{HW}}$, $F_1(\Delta\omega)$ decays exponentially and $C(\Delta\omega) = C_2(\Delta\omega) + C_3$. The full curve corresponds to $C_2(\Delta\omega)$ where $C_2(\Delta\omega) = g^{-1}F_2(\Delta\omega) = 0.09(\sin x/x)^2$ with $x = (\Delta\omega/D)^{1/2}(W/2)$ and $C_3 = g^{-2} = 0.0081$. We see that in this regime, $C_2(\Delta\omega)$ makes the dominant contribution to $C(\Delta\omega)$. The analytical result $F_2(\Delta\omega)$ was calculated

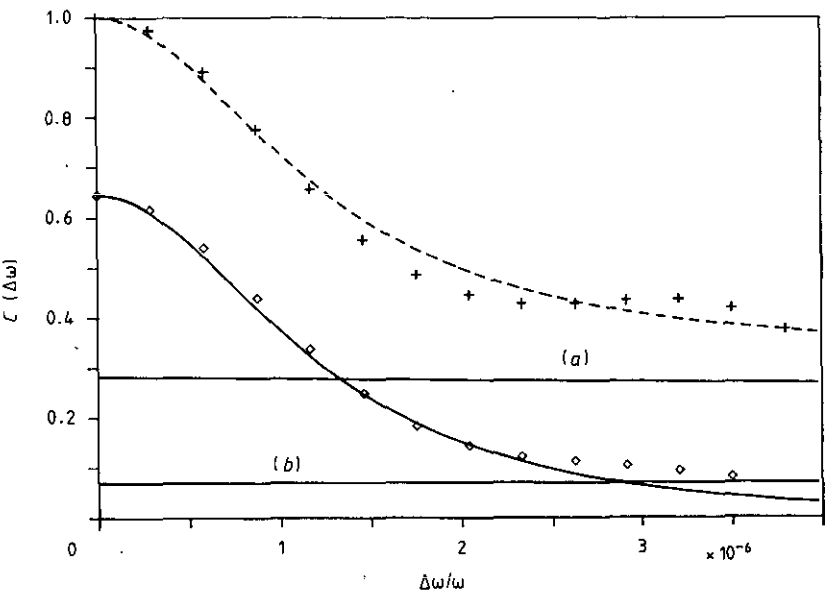

Figure 1. $C(\Delta\omega)$ as a function of $\Delta\omega/\omega$ for $L = 7W$. The squares represent the numerical results for $F_1(\Delta\omega)$ and the full curve the analytical results as extracted from (5). The plus signs represent the numerical results for $C(\Delta\omega)$ and the broken curve the analytic results (see text). Curve (a) represents $C_2(\Delta\omega)$ as given by (6) and curve (b) represents $C_3 = g^{-2}$.

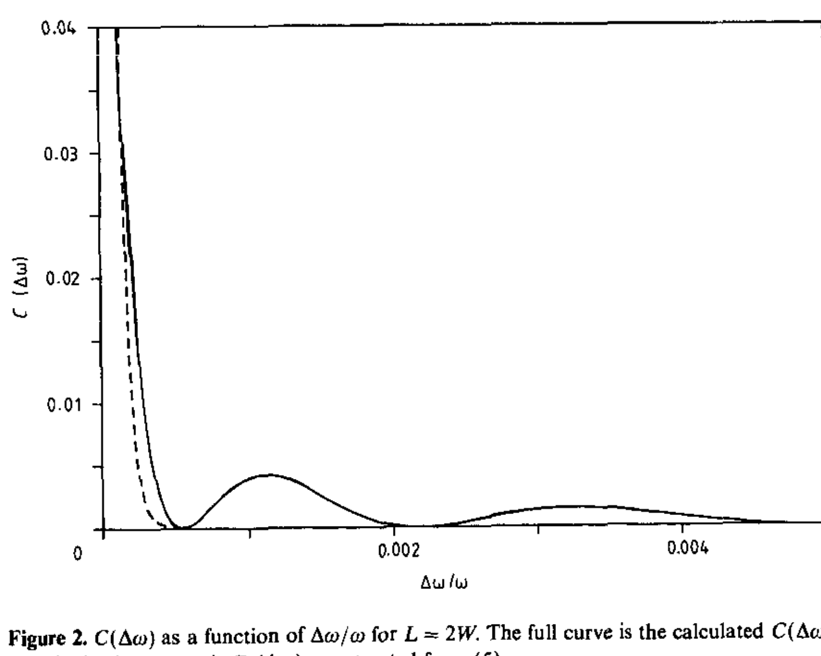

Figure 2. $C(\Delta\omega)$ as a function of $\Delta\omega/\omega$ for $L = 2W$. The full curve is the calculated $C(\Delta\omega)$ and the broken curve is $F_1(\Delta\omega)$ as extracted from (5).

as follows. By using the Hikami box diagrams [15], Eliyahu *et al* [14] have recently calculated $C_2(\Delta\bar{q}_a, \Delta\bar{q}_b)$ for a tube geometry. Setting $\Delta q_a = 0$, we get $C_2(\Delta q_a = 0, \Delta q_b) = [\sin(\Delta q_b W/2)/(\Delta q_b W/2)]^2$. When a frequency shift $\Delta\omega$ is introduced in the propagators, it serves as a cut-off and Δq_b must be replaced by $(\Delta\omega/D)^{1/2}$. This leads to (6). The numerical simulations were not accurate enough to follow this oscillatory behaviour.

In summary, we have shown that the long-range contribution $F_2(\Delta\omega)$ for a tube geometry is entirely different from that of a wide system where $W \gg L$. For tubes where $L \gg W$, $F_2(\Delta\omega)$ is a constant, independent of $\Delta\omega$ for a wide range, $\Delta\omega < D/W^2$. For $\Delta\omega > D/W^2$, $F_2(\Delta\omega)$ has an oscillatory character and dominates $C(\Delta\omega)$.

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