Effect of field tilting on the dynamics of vortices pinned by correlated disorder

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Low-temperature dynamics of flux lines in high-temperature type-II superconductors in the presence of correlated disorder in the form of columnar defects is discussed. The effect of tilting the applied magnetic field with respect to the column's directions is considered, using the non-Hermitian quantum mechanics technique used by Hatano and Nelson [N. Hatano and D. R. Nelson, Phys. Rev. Lett. **77**, 570 (1996)]. It is shown that the critical current, as well as the vortex transport properties below this current, may be determined by ''surface excitations,'' i.e., by the roughness of the flux line near the edges of the sample, which dominated the bulk jumps. Phase-space considerations determine the critical thickness of the sample, below which the tilt-induced surface transport exceeds the bulk mechanism. This critical length, which depends on the tilt angle as well as the directions of the perpendicular field and the supercurrent, diverge at the delocalization transition. [S0163-1829(97)51506-2]

Flux line response functions in cuprate high-temperature superconductors have attracted considerable interest in recent years.[1] In order to avoid dissipation of energy as a result of flux line motion driven by the superconducting current, these lines should be pinned by crystal impurities.[2] It turned out[3] that the pinning is much stronger (especially when many vortex interactions are taken into account) when these impurities are in the form of correlated disorder (such as twin boundaries[4] or columnar defects[5]) compared to the case of point disorder, resulting from vacancies of oxygen atoms.[6] However, Nelson and Vinokur[3] have pointed out that the correlated defects pinning becomes less effective in cases where the direction of the external magnetic field is tilted with respect to the defects, which we take to be along the $\hat{\mathbf{z}}$ direction. At some critical tilt, for which the energy per unit length of the defect is less than the energy associated with the perpendicular field, a pinning-depinning phase transition occurs and the flux lines delocalize.

Critical bulk current and vortex dynamics below this current for flux lines in the presence of columnar defects have been considered in Ref. 3. The authors, using the mapping of flux lines in a $d+1$ dimensional superconductor to the world lines of bosons in a d-dimensional quantum system, identified the phase-space diagram of the system which contains a high-temperature ''superfluid'' and low-temperature ''Bose glass'' phases, as well as a Mott insulator at the matching field, $B_\phi = n_{pin}\phi_0$, for which there is one flux line per defect. At low temperatures, this matching field separates the ''dilute'' region of the Bose glass phase, for which the vortex lines are pinned individually by the defects [i.e., $a_0 \approx (\phi_0/B)^{1/2}$, the Abrikosov lattice constant, is much larger than d, the typical distance between two columnar defects] from the high density region, for which interactions are important in determining the localization length and transport properties of the flux lines.

In the low-field region, the vortices are localized by the interaction with the correlated defects. Each defect is the analog of a two-dimensional (2D) potential well which we shall take (up to logarithmic corrections) as a cylindrical square well such as $V(r) = -U_0$ for $r < b_0$ and $V(r) = 0$ for $r > b_0$. The temperature of the superconductor, in turn, corresponds to the Planck constant $\hbar$ of the quantum boson problem. For the dilute vortex arrays, where the pinning energy is larger than the interaction energy, there are two regimes. For low temperature, ($T \ll T^*$, $T^* \equiv \sqrt{U_0 \epsilon_1} b_0$) the localization length $l_\perp$ is on the radius of the defect, i.e., of order b_0, so that each flux line is localized by *one* defect. On the other hand, for $T \gg T^*$, the localization length of one defect grows exponentially with T^2, and the flux line is then localized by several defects, forming an effective d-dimensional potential well in the corresponding boson system.

The response of the flux line to superconducting current in the plane perpendicular to the vortex direction $\mathbf{J} \perp \mathbf{B}$ translates itself into the response of the boson system to an applied electric field. For vortices oriented in the $\hat{\mathbf{z}}$ direction, the Lorentz force per unit length of the vortex is given by

$$\mathbf{f_L} = \frac{\phi_0}{c} \hat{\mathbf{z}} \times \mathbf{J}, \tag{1}$$

which is the analog of a boson with charge ϕ_0 interacting with the electric field $\mathbf{E} = (1/c)\hat{\mathbf{z}} \times \mathbf{J}$.

Above the critical current J_c, the vortices are no more localized and there is no superconductivity (in the sense of dissipation-free current at zero temperature) anymore. Below this critical current, the mechanism for flux flow is tunneling via thermally activated ''half loops'' (or, in the boson dynamics, tunneling into the conduction band). For currents smaller than J_1, the half-loop transverse displacement exceeds the mean distance between occupied pinning sites, and for thick samples the flux lines move via the nucleation of superkinks, the analog of the Mott variable range hopping (VRH) in doped semiconductors.[7]

The depinning of the flux line as a result of external field tilt has been carefully investigated by Hatano and Nelson.[8] The Hamiltonian of the corresponding boson problem is no longer Hermitian; the kinetic term of the Hamiltonian

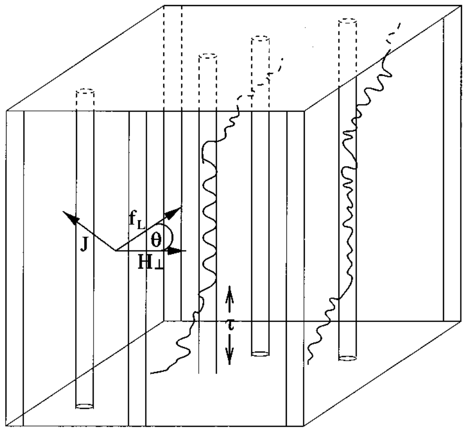

FIG. 1. Flux lines localized by columnar defects in the presence of perpendicular field $\mathbf{H}_\perp$. The surface roughness extends distance τ into the bulk, and the Lorentz force f_L is at angle θ to the tilting field. For $|\theta|>\pi/2$, the contribution of the surface roughness to the vortex transport comes from the ''tail'' at the lower end of the sample in the figure.

$\mathbf{p}^2/(2\epsilon_1)$ ($\mathbf{p}\equiv -iT\boldsymbol{\nabla}$) is subject to imaginary gauge transformation and takes the form $(\mathbf{p}+i\mathbf{h})^2/(2\epsilon_1)$, where $\mathbf{h}$ is related to the perpendicular magnetic field $\mathbf{H}_\perp$ via $\mathbf{h}=\phi_0\mathbf{H}_\perp/(4\pi)$. As a result, there are two solutions for each localized (real energy spectrum) state, corresponding to the eigenstates of the Hamiltonian and its complex conjugate. These solutions, termed ψ_R and ψ_L, correspond to the right or left ''tilting'' of the localized solution of the untilted Hamiltonian, i.e.,

$$\psi_{R,L}(\mathbf{r})=\exp(\pm\mathbf{h}\cdot\mathbf{r}/T)\psi(\mathbf{r}). \tag{2}$$

The probability distribution to find the flux line at the point $\mathbf{r}$ at a distance τ from the surface of the sample is given by

$$\begin{aligned}P(\mathbf{r},\tau)&=\mathcal{Z}^{-1}\langle\psi^f|\exp[-(L-\tau)H/T]|\mathbf{r}\rangle\\&\quad\times\langle\mathbf{r}|\exp(-\tau H/T)|\psi^i\rangle\\&=\mathcal{Z}^{-1}\sum_{m,n}\langle\psi^f|m_L\rangle\langle m_R|\mathbf{r}\rangle\langle\mathbf{r}|n_L\rangle\\&\quad\times\langle n_R|\psi^i\rangle e^{-[\tau E_m+(L-\tau)E_n]/T},\end{aligned} \tag{3}$$

such that, as $L\to\infty$, the probability distribution of the flux line at the surface is proportional to $\langle\mathbf{r}|gs_{L,R}\rangle=\psi_{gs}^{L,R}(\mathbf{r})$ where $|gs_{L,R}\rangle$ are the left and the right ground state, respectively. Deep in the bulk, the distribution is given by $P(\mathbf{r},L/2)=\psi_{gs}^R\psi_{gs}^L=\psi_{untilted}^2$, i.e., in the localized regime, the flux line changes its shape near the surface, while remaining unaffected in the bulk (see Fig. 1). Typically, the ''surface roughness'' associated with the tilt extends into the bulk up to some characteristic distance τ^*, which diverges as the tilting angle approaches the critical angle, for which the flux line delocalizes and the current response becomes linear.

In this paper, we study the the effect of the tilt on the flux line response to superconducting currents in the regime where the tilting angle is smaller than critical, i.e., in the Bose glass phase where the flux lines are localized. We assume that the thickness of the sample is large enough, such that it is much larger than the dimension of the optimal excitation along the $[\hat{\mathbf{z}}]$ axis. Moreover, we address only the dilute limit, for which the transverse (xy) displacement of these excitations is less than a_0, so that the interaction is taken into account by filling up the localized states in order of increasing energy up to the chemical potential μ.

Let us consider first the critical current. This current is determined by the binding-free energy $U(T)$ as well as the localization length $l_\perp=1/\kappa$. Modeling the defect as a square potential well in the boson system, κ is related to $U(T)$ by $\kappa=\sqrt{2U(T)\epsilon_1}/T$. Of these two, $l_\perp$ is changed as the magnetic field is tilted. Near the surface of the sample, the localization length should be $l_\perp(h,\theta)=1/[\kappa-h|\cos(\theta)|T]$, where θ is the angle between the Lorentz force f_L (perpendicular to the supercurrent $\mathbf{J}$, which we take to be in the xy plane) and the tilting field $\mathbf{h}$. The absolute value is needed for the case of $|\theta|\rangle\pi/2$, for which the critical current is dominated by the ''tail'' of the flux line on the other end of the sample, as shown in Fig. 1. Thus, the critical current will take the form

$$J_c(T,h,\theta)=\frac{cU(T)[T\kappa(T)-h|\cos(\theta)|]}{T\phi_0}. \tag{4}$$

This critical current is determined by the surface ends of the vortex, for which the effect of the tilt is maximal. However, the ''creep'' of the vortex in the direction of the tilting field is limited by the effect of ''image vortices'' which should be introduced in order to satisfy the boundary conditions on the surface.[9] These image vortices will lock the flux line to the defect and cancel the effect of the tilt in the region determined by the London penetration depth λ near the surface. Thus, for very small perpendicular magnetic fields, where the surface roughness extension τ^* is less than the London length, the tilt has no effect on the vortex pinning and there is no change in the response functions of the flux system.

For currents less than critical, the thermally assisted flux flow (TAFF) theory of the vortex transport gives the resistivity $\rho=\mathcal{E}/J$ as

$$\rho=\rho_0e^{\delta F/T}, \tag{5}$$

where δF is the energy barrier for flux line jumps. Our basic observation is that deep in the bulk there is no influence of the tilt, so that the energy barriers for nucleating half-loops or double kinks are the same. The physical reason for it is that, although the perpendicular field *decreases* the energy barrier for one side of the kink/loop, it *increases* the energy needed for the other side. The main effect of the tilt comes from *surface kinks/loops*, for which the energy barrier really decreases. Although the resulting free-energy barrier δF is smaller than the bulk one, so that the ''resistivity'' associated with it is exponentially smaller, one should take into account the phase space prefactor of these two mechanisms — the number of surface kinks available is determined by the width of the surface roughness, i.e., by τ^*, while the number of bulk kinks is of order $(L-\tau^*)/Z$, where Z is the distance for which the half loop/kink extends along the relevant defect. Thus the nature of the current response is determined by the thickness of the sample — for $L>L_c(\mathbf{h})$, the bulk excitations will dominate and the response is tilt independent, while for $L<L_c$ surface excitations become important and the voltage drop will be tilt dependent. As $h\to h_c$ (where h_c is the critical field above which the flux lines

delocalize) L_c diverges, so that near the depinning transition the resistivity of the system goes continuously to zero.

In order to estimate the relevant quantities in the tilted case, we use the expressions for the free energy of the surface excitations in the presence of the tilt. Consider now surface excitation of the flux line with line tension ϵ_1 which extends for a distance z along the pin and has perpendicular extent r. The free energy of such jumps is given by

$$\delta F=\frac{\epsilon_1 r^2}{z}+U_0 z-f_L rz-hr\cos(\theta) \tag{6}$$

for the ''half loop'' surface excitations. If the jump is due to the nucleation of superkinks, one should take into account the energy differences between different rods at distance r. This, in turn, is determined by the density of states at the chemical potential $g(\mu)$,[3] and the free energy is

$$\delta F=2E_k\frac{r}{d}+\frac{z}{g(\mu)r^2}-f_L rz-hr\cos(\theta). \tag{7}$$

The resulting saddle-point free energies are

$$\delta F^*=[E_k-h|\cos(\theta)|d](J_1/J) \quad \text{half loops} \tag{8}$$

$$\delta F^*=(E_k-h|\cos(\theta)|d)(J_0/J)^{1/3} \quad \text{superkinks,} \tag{9}$$

where $E_k=\sqrt{\epsilon_1 U_0}d$, $J_1=cU_0/(\phi_0 d)$, and $J_0=c/[\phi_0 g(\mu)d^3]$, for d the average spacing between unoccupied pins.

Let us estimate now the phase space for such surface excitations, i.e., the width of the region in which this roughness takes place. Using Eq. (3) one finds that the crossover between the surface $[P(\mathbf{r})\sim\psi^{R,L}(\mathbf{r})]$ and the bulk, for which $P(\mathbf{r})$ is the same for the tilted and the untilted situation, is determined by the quantity

$$Y(\mathbf{r})=\sum_m \langle m_L|[r]\rangle\exp(-\tau E_m/T), \tag{10}$$

where τ is the distance from the surface. The transition to the surface behavior takes place when $Y(\mathbf{r})$ becomes $\mathbf{r}$ independent, and thus absorbed into the normalization factor for $P(\mathbf{r})$. Typically, this happens when Eq. (10) is not dominated by the τ-dependent exponential factor, since then the summation over m is determined by the delocalized states, yielding an $[r]$ independent result. Thus, for the case of half-loop tunneling, the width of the surface roughness will be $\tau^*(h,\theta)\approx T/E^*(h,\theta)$ where $T\kappa(E^*)=h$. This gives us the estimate

$$\tau^*_{loops}=\frac{T\epsilon_1}{(h_c^2-h^2)}. \tag{11}$$

For the superkinks tunneling, the energy E_m may be given by $1/[g(\mu)r_m^2]$, so that the energy exponent becomes negligible as $\tau<\tau^*$, where[9]

$$\tau^*=\frac{T^3 g(\mu)}{(h_c-h)^2}. \tag{12}$$

The phase space of the surface excitations is given by the width of the surface region divided by the ''width'' of the typical excitation, $z^*_{surface}$. Using the above expressions for the free energy of the kinks/loops, one finds that

$$z^*_{surface,loops}(J,h,\theta)=\frac{c[\sqrt{\epsilon_1 U_0}-h|\cos(\theta)|]}{J\phi_0} \tag{13}$$

and

$$z^*_{surface,kinks}(J,h,\theta)=\frac{c[E_k-h|\cos(\theta)|]}{J\phi_0 d} \tag{14}$$

The resulting resistivity in thick samples will be determined by adding in parallel the τ^*/z^* ''surface resistors'' with $\rho=e^{\delta F^*_{surface}/T}$ to the system of $(L-\tau^*)/Z^*_{bulk}(J)$ ''bulk resistors'' with $Z^*_{bulk}(J)=z^*_{surface}(J,h=0)$. While the surface roughness does not depend on the angle between the current and the transverse magnetic field, the width of the jump, as well as the free-energy barrier, do depend on it. It turns out that the resistivity in the ''perpendicular'' direction ($\mathbf{f_L}\perp\mathbf{h}\perp[z]$) is independent of the tilt. For other directions of the superconducting current, there will be a crossover length L_c below which the surface loops dominate the jumps. For any tilt less than critical, the surface roughness is finite, so that as $L\to\infty$, bulk excitations are clearly the preferred jumping mechanism, but as $h\to h_c$, the width of the surface roughness becomes comparable with the sample thickness for each finite sample and one sees a crossover to surface-excitations-dominated transport. The critical length is related to the parameters above as

$$L_c(J,h,\theta)=\tau^*\frac{Z^*_{bulk}}{z^*_{surface}}\exp(\delta F^*_{bulk}-\delta F^*_{surface}). \tag{15}$$

There are two reasons for the divergence of L_c as $h\to h_c$; one is the divergence of τ^*, the other is the fact that $z^*_{surface}\to 0$, yielding infinite phase space for the surface excitations. However, there is a limitation on the minimal width z of the jumps; as $z\to\lambda$, the London length, self-interaction of the flux line locks the kink/loop, so that λ sets the minimal excitation extent along the $\hat{\mathbf{z}}$ axis. For the region in parameter space for which $z^*\gg\lambda$, the critical length will grow as $[h|\cos(\theta)|-h_c]^{-2}$ for loop transport, and as $[h|\cos(\theta)|-h_c]^{-3}$ for kinks. On the other hand, as one approaches the critical tilt, the region

$$[h_c-h|\cos(\theta)|]<\frac{\lambda J}{c\phi_0} \tag{16}$$

is entered, in which the excitations width could not shrink anymore. In that case the critical length diverges as $[h|\cos(\theta)|-h_c]^{-1}$ for loops, and as $[h|\cos(\theta)|-h_c]^{-2}$ for kinks.

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[1] See, e.g., G. Blater, M. V. Feigelman, V. B. Geshkenbein, A. I. Larkin, and V. M. Vinokur, Rev. Mod. Phys. **66**, 1125 (1994), and references therein.
[2] P. W. Anderson, Phys. Rev. Lett. **9**, 309 (1962).
[3] D. R. Nelson and V. M. Vinokur, Phys. Rev. B **48**, 13 060 (1993).
[4] W. K. Kwok, S. Fleshler, U. Welp, V. M. Vinokur, J. Downey, G. W. Crabtree, and M. M. Miller, Phys. Rev. Lett. **69**, 3370 (1992).
[5] M. Konczykowski *et al.*, Phys. Rev. B **44**, 7167 (1991); L. Civale *et al.*, Phys. Rev. Lett. **67**, 648 (1991).
[6] M. Tinkham, Helv. Phys. Acta **61**, 443 (1988).
[7] B. I. Shklovskii and A. L. Efros, *Electronic Properties of Doped Semiconductors* (Springer-Verlag, New York, 1984).
[8] N. Hatano and D. R. Nelson, Phys. Rev. Lett. **77**, 570 (1996).
[9] See, e.g., V. Ambegaokar, B. I. Halperin, D. R. Nelson, and E. D. Siggia, Phys. Rev. B **21**, 1806 (1980).